

Fractal Dynamics Of EEG Signals: Julia Set Perturbations In Mild Cognitive Impairment

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Abstract

Mild Cognitive Impairment (MCI) is a transitional stage between normal aging and dementia that increases the risk of Alzheimer’s disease. This study investigated the sensitivity of Julia fractals to perturbations induced by electroencephalographic (EEG) and electrooculographic (EOG) signals from older adults with normal cognition and MCI. Resting-state recordings were segmented into consecutive 20-second intervals, and relative alpha-band power was estimated as a marker of cognitive activity. The normalized signals were used to perturb a quadratic Julia map, generating fractal structures whose complexity was quantified using the box-counting dimension. Significant differences in fractal dimension were identified between cognitively normal and MCI participants across multiple EEG channels, particularly in frontal, temporal, parietal, and central regions. These findings suggest that EEG-driven Julia fractals capture nonlinear alterations associated with cognitive impairment and may provide a noninvasive framework for the early detection of MCI.

Introduction

Mild Cognitive Impairment (MCI) is considered an intermediate stage between normal aging and dementia [1]. In Mexico, it affects approximately 7.3% of the adult population and may increase the risk of developing dementia by up to 46% within a three-year period [2].

Electroencephalographic (EEG) signals exhibit well-established fractal properties that reflect the intrinsic complexity of neuronal activity and large-scale brain dynamics [3, 4, 5]. Alterations in these nonlinear characteristics have been associated with cognitive decline and neurodegenerative processes. In particular, changes in alpha-band activity (8–13 Hz), which is closely related to attention and resting-state cognitive function, have been consistently reported in individuals with cognitive impairment [6].

In this work, we propose the use of Julia fractals perturbed by physiological time series as a novel mathematical framework for identifying quantitative biomarkers of MCI. This approach combines non-linear dynamical systems with physiological recordings to investigate subtle changes in brain function. Julia sets describe the behavior of complex numbers under the iteration of nonlinear functions. In particular, we consider the quadratic family generated by

$$Z_{n+1} = Z_n^2 + c, \quad (1)$$

where $c \in \mathbb{C}$. The resulting dynamics strongly depend on this parameter, producing highly sensitive fractal structures that may exhibit connected or disconnected geometries. Such sensitivity makes Julia fractals suitable for exploring perturbations induced by physiological signals.

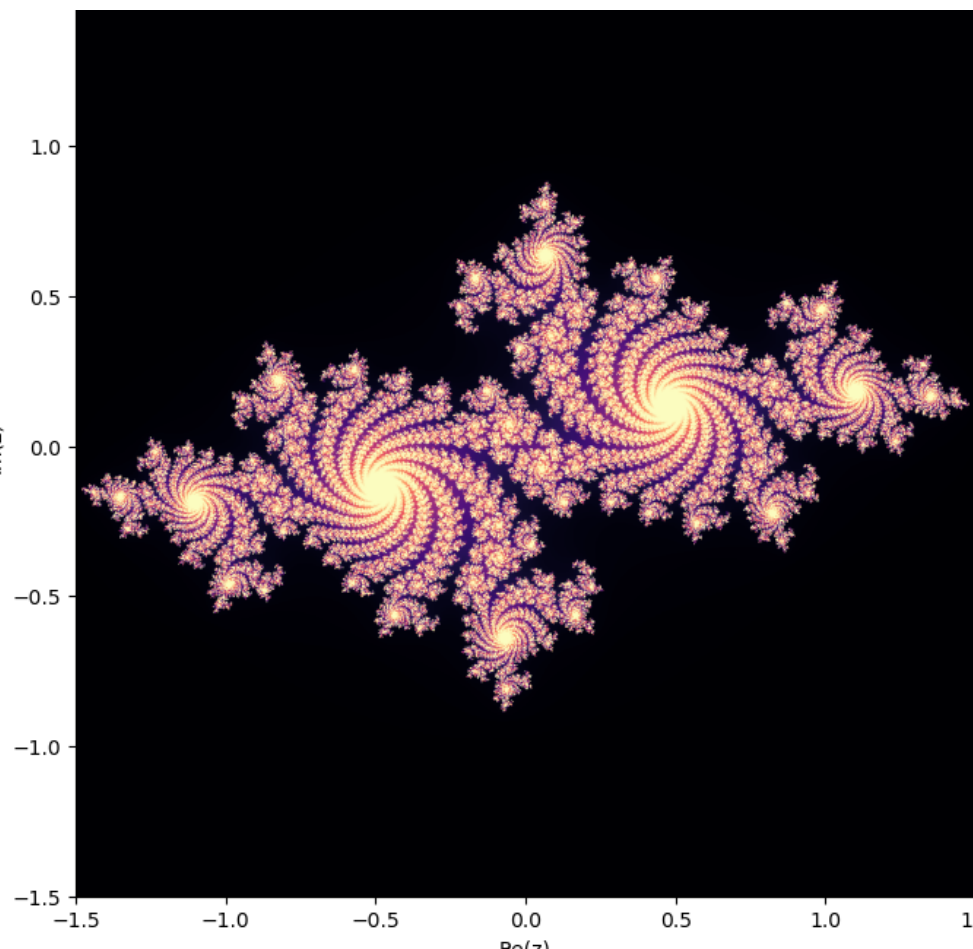


Figure 1: Julia fractal with $c = -0.7 + 0.27015i$.

Results

Julia fractals were generated using values of c distributed along a parametric line. Ten consecutive 20-second segments per participant were analyzed, applying the perturbation defined in Equation (3) to assess intra-subject variability and compare fractal properties between Control and MCI groups. Significant differences in fractal dimension ($p < 0.05$) were observed, with MCI participants consistently showing higher values, indicating increased geometric complexity. These effects were present across frontal, central, parietal, and temporal regions, with the strongest difference at T4 ($p = 0.012$). Representative cases from C4 and P3 revealed distinct fractal morphologies associated with group differences.

EEG Channel	Interval	$\bar{D}_{\text{Ctrl}}$	$\bar{D}_{\text{MCI}}$	p-value	Statistical Test
C3	8	1.251	1.625	0.043	Mann–Whitney U
C4	2	1.130	1.478	0.048	Student’s t -test
F3	5	1.197	1.514	0.042	Student’s t -test
P3	3	1.122	1.473	0.046	Student’s t -test
P4	1	1.146	1.412	0.043	Mann–Whitney U
T4	1	1.174	1.631	0.012	Mann–Whitney U
T5	1	1.213	1.579	0.027	Mann–Whitney U

Table 1: Differences in fractal dimension of EEG-perturbed Julia sets between Control and MCI participants. $\bar{D}_{\text{Ctrl}}$ and $\bar{D}_{\text{MCI}}$ denote the mean fractal dimensions for each group. Statistical tests were selected based on normality (Shapiro–Wilk) and variance homogeneity (Levene), with significance set at $p < 0.05$.

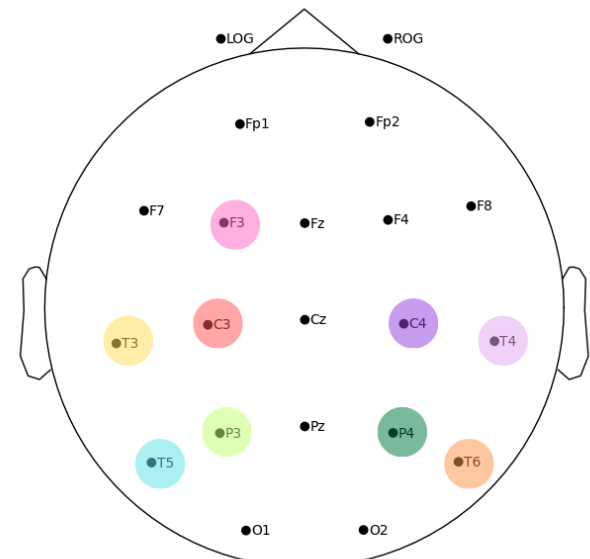


Figure 2: Significant differences were observed in the highlighted channels.

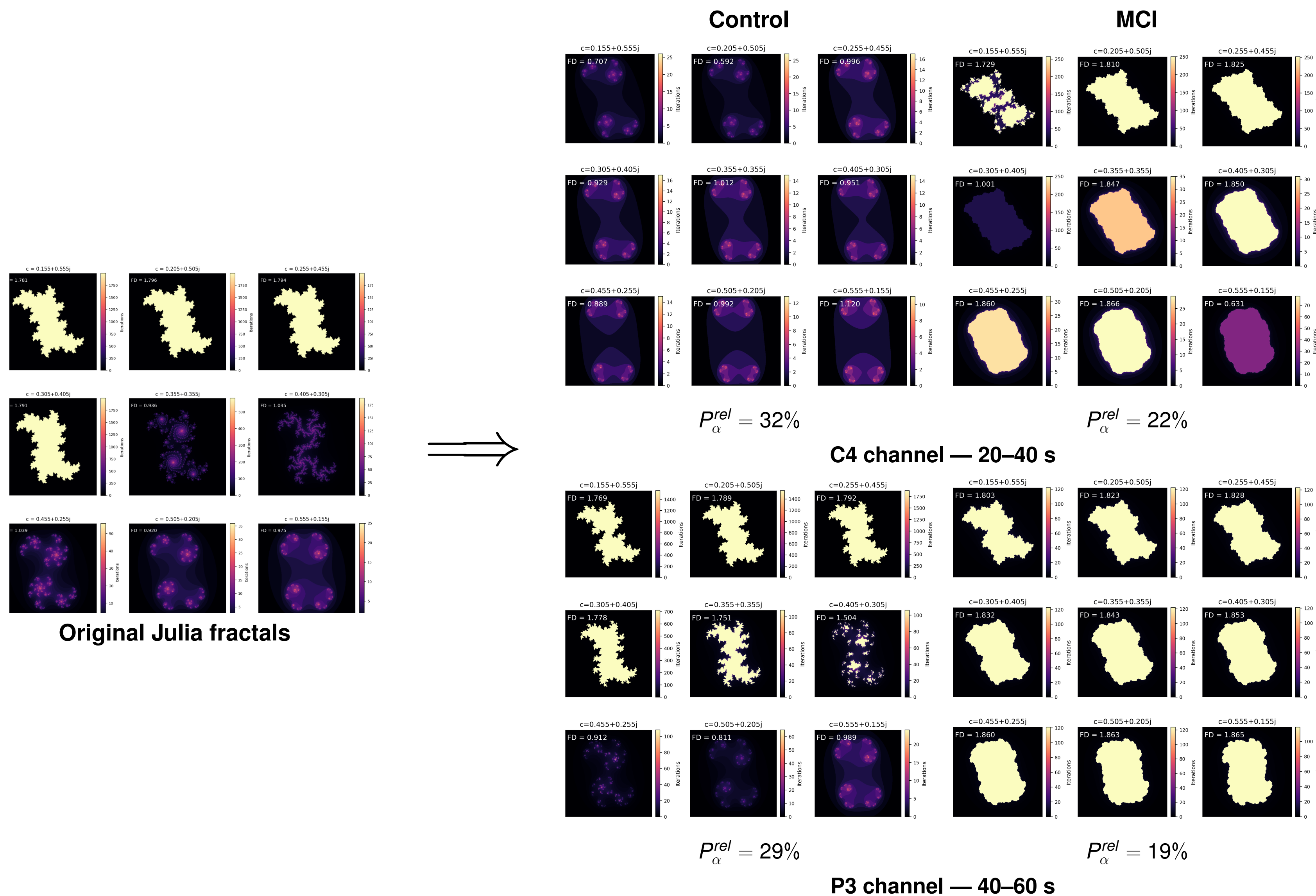
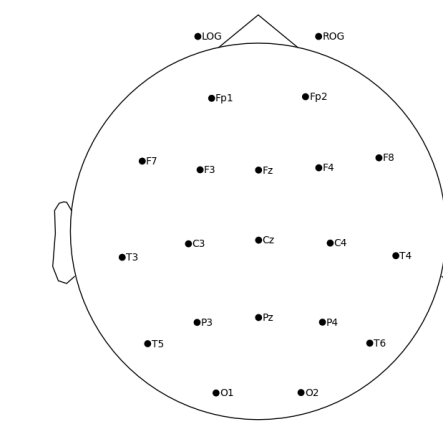


Figure 3: Representative examples from channels C4 and P3 reveal distinct fractal morphologies between groups. Reduced alpha-band activity was associated with increased fractal complexity.

Methods

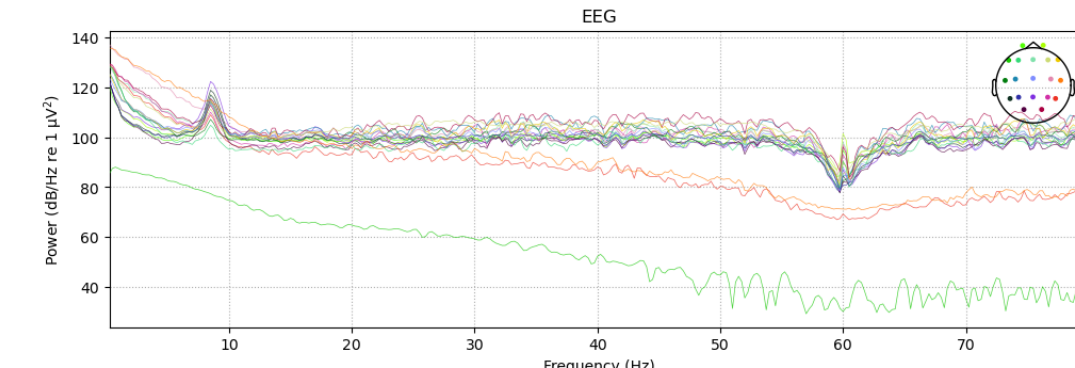
Data Acquisition

Resting-state EEG (19 channels) and EOG recordings were obtained from 18 older adults (66 ± 6 years) under eyes-closed conditions. Cognitive status was evaluated using the standardized **NEUROPSI** assessment, classifying participants as Controls ($n = 10$) or Mild Cognitive Impairment (MCI, $n = 8$).



Signal Preprocessing

Signals were processed using **MNE-Python**. Band-pass filtering was applied and artifacts were removed through visual inspection. Clean recordings were segmented into consecutive **20-second windows**.



Alpha-Band Analysis

Power spectral density (PSD) was estimated using Welch’s method. Relative alpha power (8–13 Hz) was computed as:

$$P_{\alpha}^{rel} = \frac{P_{8-13\text{ Hz}}}{P_{total}}, \quad (2)$$

where P_{total} denotes the total spectral power within the analyzed frequency range. This metric quantifies the strength of alpha-band oscillatory activity associated with cognitive function.

EEG-Driven Julia Fractals

Each EEG and EOG signal (X_n) was normalized to the interval $[-1, 1]$ and introduced as a perturbation into the Julia map:

$$Z_{n+1} = Z_n^2 + c + k X_n, \quad (3)$$

where $c \in \mathbb{C}$ and k controls the perturbation intensity. In this study, we set $k = 0.5$, thereby generating subject-specific Julia sets.

Fractal Complexity

The geometric complexity of each perturbed Julia set was quantified using the **box-counting dimension**. The fractal dimension was estimated from the slope of the linear scaling relationship between $\log N(\epsilon)$ and $\log(1/\epsilon)$, given by

$$D_f = \lim_{\epsilon \rightarrow 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)}. \quad (4)$$

Statistical Evaluation

Group comparisons (Control vs MCI) were performed using Shapiro–Wilk for normality and Levene’s test for homogeneity. Depending on assumptions, Student’s t -test or Mann–Whitney U test was applied. Significance was set at $p < 0.05$.

Conclusions

Julia fractals perturbed by EEG and EOG signals capture nonlinear alterations associated with mild cognitive impairment. This methodology is sensitive to regional differences in neural dynamics and represents a complementary, noninvasive framework for developing quantitative markers of early cognitive dysfunction.

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AAIC 26

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